Squeezing Behaviour of a Degenerate Paramp with Fourth-Order Interaction: Stationary and Transient Statistics

C. Cabrillo[1], *P. García Fernández*[1], *F.J. Bermejo*[1], *P. Colet*[2], *R. Toral*[2], *and M. San Miguel*[2]

[1]Instituto de Estructura de la Materia, C.S.I.C.,
Serrano 119, E-28006 Madrid, Spain
[2]Departament de Física, Universitat de les Illes Balears,
E-07071 Palma de Mallorca, Spain

Abstract. A model system representing a degenerate parametric amplifier plus a fourth-order term (Kerr effect) is analyzed. Such higher-order interaction leads to the appearance of a pitchfork bifurcation showing bright-squeezing in the stationary states. A numerical simulation of the transient processes is presented and the transient-squeezing characteristics are finally discussed.

1 Introduction

It has been shown that [1] an intense coherent beam which propagates along an optical Kerr medium, exhibits "self- squeezing" (amplitude squeezing), and that the effect could be remarkably high provided that the incident pump strength is high enough. The effect of such higher-order interactions coupled to a standard two-photon medium was analyzed by Tombesi [2] some time ago. The interest in the already mentioned study was focused on the possibilities of generating strong squeezed light at short interaction times (i.e., in travelling-wave geometries). Our interest here is to explore the field- statistical properties of a more realistic model of a parametric amplifier where the crystal losses are explicitly accounted for. The effective Hamiltonian, the obtention of the semiclassical equations and the stability analysis have been presented in an accompanying paper [3].

2 Fluctuations around steady-states

As it has been shown [3], the stochastic equations for the quadrature fluctuations around the stable solutions are

Springer Proceedings in Physics, Vol. 55
Nonlinear Dynamics and Quantum Phenomena in Optical Systems
Editors: R. Vilaseca, R. Corbalán

$$\begin{aligned}\delta\dot{x} &= (\mu + 2\nu\bar{y}_S\bar{x}_S)\delta x + \nu(\bar{x}_S^2 + 3\bar{y}_S^2)\delta y + L_x \\ \delta\dot{y} &= -\nu(3\bar{x}_S^2 + \bar{y}_S^2)\delta x - (1 + 2\nu\bar{x}_S\bar{y}_S)\delta y + L_y\,. \end{aligned} \tag{1}$$

The stationary covariance matrix

$$\sigma_{ij} \equiv < x_i x_j > - < x_i >< x_j > \tag{2}$$

can be obtained from the solution of [4]

$$A\sigma + \sigma A^T = -\bar{D}I, \tag{3}$$

where A is the drift matrix (expressed in terms of the coefficients given in eq. (1)) and I is the unit matrix. In the case below threshold, the mean square fluctuations in the two quadrature components are

$$< x^2 >= \frac{\gamma(1+2\bar{n})}{4(\gamma-\kappa)}\,, < y^2 >= \frac{\gamma(1+2\bar{n})}{4(\gamma+\kappa)}\,. \tag{4}$$

The zero-point fluctuations are $< x^2 >=< y^2 >= 1/4$, so that, for sufficiently low temperature ($\bar{n} \to 0$), the y-component is squeezed. Since the fluctuations are about the steady-state value $(\bar{x}_S, \bar{y}_S) = (0,0)$, this represents a squeezed vacuum state. The product of the mean squares is in this case

$$< x^2 >< y^2 >= \frac{1}{16}\frac{\gamma^2}{\gamma^2-\kappa^2}(1+2\bar{n}^2)\,, \tag{5}$$

that is greater than 1/16. Thus, the state is not a minimum uncertainty state.

Above threshold a pitchfork bifurcation appears leading to two steady-state solutions (bistability, [3]) which are

$$\begin{aligned}\bar{x}_S &= \mp[\mu^{\frac{1}{2}}/\nu(1+\mu)]^{\frac{1}{2}} \\ \bar{y}_S &= -\mu^{\frac{1}{2}}\bar{x}_S\,. \end{aligned} \tag{6}$$

Making use of eq. (2) and diagonalizing the covariance matrix, the stationary variances become

$$\begin{aligned}\sigma_{11} &= \frac{\bar{D}}{16(1-\mu)\mu(1+\mu)^2}[B + (\mu^2-1)B^{1/2}] \\ \sigma_{22} &= \frac{\bar{D}}{16(1-\mu)\mu(1+\mu)^2}[B - (\mu^2-1)B^{1/2}] \\ B &= \mu^4 + 16\mu^3 + 30\mu^2 + 16\mu + 1. \end{aligned} \tag{7}$$

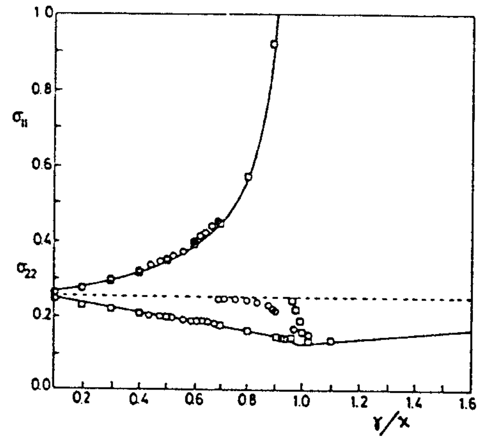

Fig. 1: Comparison between the stationary variances numerically determined (○ correspond to $\Gamma/\kappa = 2\times10^{-2}$, □ to $\Gamma/\kappa = 2\times10^{-3}$) and the variances determined from the linearized theory (solid line).

As it is shown in fig. 1 the 22-component is squeezed. Note that since $(\bar{x}_S, \bar{y}_S)$ are real and different from zero, "bright squeezing" appears.

3 Analysis and discussion of the results

3.1 Stationary behaviour

Besides the linearized analysis of the Langevin equations a numerical approach has been carried out. Both kinds of results are shown in fig. 1. It can be easily seen that for regions far away from threshold, the linearized theory is in good agreement with the simulation results. However, for some values the linearized theory predicts maximum squeezing whereas a substantial increase in σ_{22} is found in the simulation. Such a fact arises from the bistable behaviour of this system. As can be seen [3] the nontrivial solutions are located at points $(\bar{x}_S, \bar{y}_S)$ depending upon the values of Γ/κ and γ/κ. Therefore their relative separation can be controlled by a suitable choice of Γ/κ. For values of $\Gamma/\kappa \leq 2 \times 10^{-3}$ and $\gamma/\kappa = 0.9$, no jumps between the stable states occur in observable time. For larger values of Γ/κ the probability of hopping attains larger values thus leading to an increase in σ_{22}.

3.2 Transient behaviour

We have analyzed the transient behaviour by means of a numerical procedure. All the obtained curves are for $\bar{n} = 0$ and $\kappa = 0.5$. The case below threshold is shown in figure 2. The figures 3 and 4 display the results for different parameters and initial condition values. It is worth noting that the stationary values are greater than those of the previous analysis. This is due to the fact that the numerical analysis is performed as an average over trajectories and bistability forces,that about half of trajectories go to one of the stationary states while the rest go to the other. The main result common to all curves is the existence of "over-squeezing" with respect to the stationary value. Figure 3 shows the curves which correspond to a coherent initial state and to a squeezed initial state. No dependence upon the initial state in σ_{22} value during "oversqueezing" is observed, i.e., after a certain time the transient evolution loses the memory about the initial conditions. The time where "oversqueezing" occurs is larger, as could be expected, for a initial coherent vacuum.

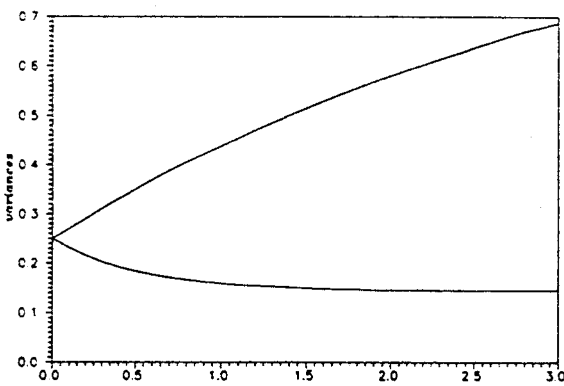

Fig. 2: Transient variances below threshold.

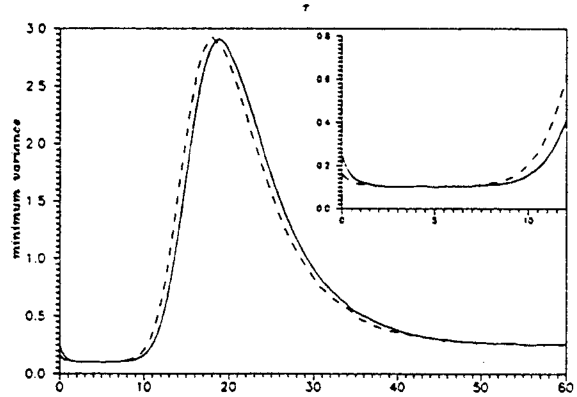

Fig. 3: Minimum transient variances above threshold (solid line coherent initial state, dashed line $<(\Delta x)^2>=1/2$, $<(\Delta y)^2>=1/6$ squeezed initial state. Both $\Gamma/\kappa = 2\times 10^{-3}$, $\gamma/\kappa = 0.75$).

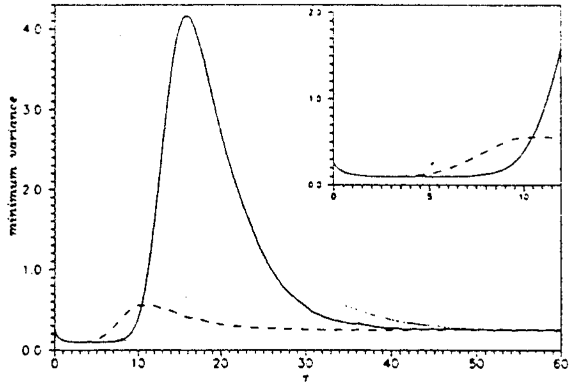

Fig. 4: Minimum transient variances above threshold(solid line $\Gamma/\kappa = 2\times 10^{-3}$, $\gamma/\kappa = 0.64$, dashed line $\Gamma/\kappa = 2\times 10^{-2}$, $\gamma/\kappa = 0.64$, pointed line $\Gamma/\kappa = 2\times 10^{-3}$, $\gamma/\kappa = 0.75$. All with coherent initial states).

Figure 3 shows the dependence with respect Γ/κ and γ/κ. The time-lag where "oversqueezing" occurs is larger for $\Gamma/\gamma = 2\times 10^{-3}$, as it could be expected since the nontrivial stationary states are now far from the origin. On the other hand, as is shown in the figure,such a time-lag becomes larger as one approaches the threshold (i.e., larger for $\gamma/\kappa = 0.64$ than for 0.75).

Finally it is worth remarking that the σ_{22} value in the "oversqueezing" region does not show any noticeable dependence on Γ/κ or γ/κ.

4 Conclusions

The effect of a fourth-order (Kerr-effect) non-linearity in a degenerate parametric amplifier has been analyzed. From the analysis of the semi-classical equations it has been found that maximum squeezing occurs in the vicinity of the threshold. On the other hand, the bistable behaviour encountered above threshold may provide a means for the generation of bright-squeezed light. Some care should be taken with respect to the results obtained near threshold since: a) the model herein analyzed constitutes an oversimplification of a real device (undepleted pump), and b) the full quantum-dynamics has not been explored, and therefore, the estimated jump rates between the two stable states should be taken as semiquantitative [5].

A substantial "oversqueezing" has been found when analyzing the transient dynamics. The time duration where "oversqueezing" occurs is enlarged near threshold. On the other hand, the best results are obtained for small values of Γ/κ.

The obtained results may therefore indicate a way for the generation of bright-squeezed light in travelling-wave devices.

References

[1] R. Tanas, S. Kielich, Quantum Opt. 2, 23, (1990).

[2] P. Tombesi in "Quantum Optics IV", J.D. Harvey, D.F. Walls (Eds.), Springer, 1986, p.81.

[3] F.J. Bermejo, P. García–Fernández, P. Colet, S. Bal.le, R. Toral, M. San Miguel (this volume).

[4] C.W. Gardiner, "Handbook of Stochastic Methods", Springer, Berlin (1985), Chap 4.

[5] P.D. Drummond, P. Kinsler, Phys. Rev. A40, 4813, (1989).