GAIN NOISE IN DYE LASERS

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INTRODUCTION

Dye laser light exhibits anomalous statistical properties. The current theoretical model[1] of dye laser fluctuations includes pump fluctuations with a finite correlation time (colored noise). This standard model is obtained in an approximation which neglects fluctuations of the saturation term. The term neglected describes fluctuations in the gain parameter. The approximation is formally equivalent to consider fluctuations in the loss parameter . In this sense we refer to the standard model as loss-noise model, as opposed to the more fundamental gain-noise model, which we study here. Experimental evidence of colored gain-noise fluctuations has been reported[2]. A first understanding of the differences between gain and loss noise models can be obtained considering white noise fluctuations[3]. We find that a white gain-noise model already describes correctly anomalous intensity fluctuations and a first order-like transition for the most probable intensity value. An initial slow decay of the intensity correlation function can also be obtained within this model. However, the relaxation involving two-time scales which is observed experimentally can not be described by a white noise model. Instead, it requires colored noise modeling. Here we present a study of intensity correlation functions in the case of colored noise. A linearized analysis identifies that differences between the normalized intensity correlation functions of colored loss and gain-noise models associated with the correlation time of the noise τ are noticeable when the cavity decay rate and τ^{-1} become comparable. We report here numerical calculations in which such differences are evidentiated in appropriate ranges of parameters .

Our gain-noise model for a single mode dye laser in resonance is defined by the following stochastic equation for the intensity:

$$d_t I = 2(-\kappa+\Gamma/(1+\beta I))I + D + (I/(1+\beta I))\,(2Q)^{1/2}\,\xi(t) + (2DI)^{1/2}\,q(t) \qquad (1)$$

where κ is the loss parameter, Γ the gain parameter and β a positive parameter involving the matter-radiation coupling constant and the polarization and population inversion decay rates. The process q(t) models spontaneous emission noise of strength D. The random force $\xi(t)$ models fluctuations of the gain parameter of strength Q. The spontaneous emission noise is taken to be Gaussian white noise of zero mean and correlation $\langle q(t)\, q(t')\rangle = 2\delta(t-t')$. The gain noise is also taken to be Gaussian of zero mean but with a finite correlation time τ : $\langle \xi(t)\, \xi(t')\rangle = \tau^{-1} \exp(-|t-t'|/\tau)$.

Coherence and Quantum Optics VI
Edited by J. H. Eberly *et al.*
Plenum Press, New York, 1990

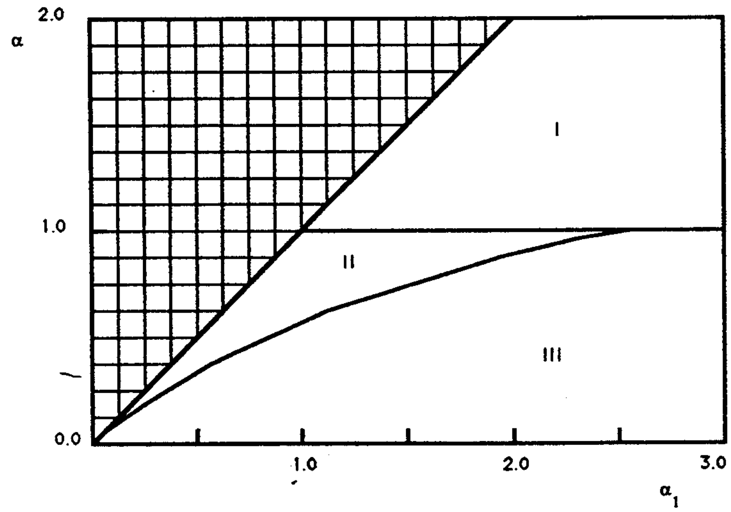

Fig. 1. Parameter plane α_1, α . Regions I, II and III correspond to differents shapes of Pst(I), as explained in the text.

WHITE GAIN NOISE

We first study the limiting case of white gain noise in which τ=0. We consider situations above threshold where spontaneous emission noise is known to have negligible influence: Then we can further simplify the problem taking D=0. In such a case the model has two independent parameters which can be chosen as $\alpha_1 = (\Gamma / Q)$, $\alpha_2 = (\kappa/Q)$. The corresponding white-noise version of the standard loss-noise model has only one independent parameter. In this convenient parametrization the standard loss-noise model is recovered from the gain-noise model in the limit $\alpha_1 \to \infty$ with $\alpha = \alpha_1 - \alpha_2$ fixed.

The stationary solution of the Fokker-Planck equation obtained for τ=0 from (1) for the intensity distribution can be found analytically. The intensity fluctuations are well described by both the gain and loss-noise models. The analysis of the extrema of the stationary intensity distribution $P_{st}(I)$ indicates the existence of three different regions in parameter space (see Fig.1): In region I (α>1), a single maximum at $I \neq 0$ exists. In region II, $P_{st}(I=0)=\infty$, and a relative maximum and minimum exist. In region III, $P_{st}(I)$ decreases monotonously with the intensity. The mean intensity grows when decreasing α_1 at α fixed or when increasing α at α_1 fixed. In the second case, the most probable intensity changes discontinuosly at α=1 regardless of the value of α_1 if 2.53 > α_1 >1. This change becomes continuous at the same point α=1 for α> 2.53. The experimental finding of this discontinuity[4] and of the presence of relative extrema in $P_{st}(I)$ was interpreted as evidence of the existence of colored noise fluctuations. Our results indicate that both effects can be consisently described within a gain-noise model which only includes white noise.

The early time decay of the normalized steady state intensity correlation function $\lambda(t) = (\langle I(t'+t)\, I(t')\rangle - \langle I\rangle^2) / \langle I\rangle^2$ can be described in terms of an effective eigenvalue λ_{eff} proportional to the initial slope of $\lambda(t)$. In our case, the explicit form for λ_{eff} is, for D=0, and τ =0:

$$\lambda_{eff} = \frac{2\,Q\langle (I/(1+\beta I))^2 \rangle}{\langle I^2 \rangle - \langle I \rangle^2} \qquad (2)$$

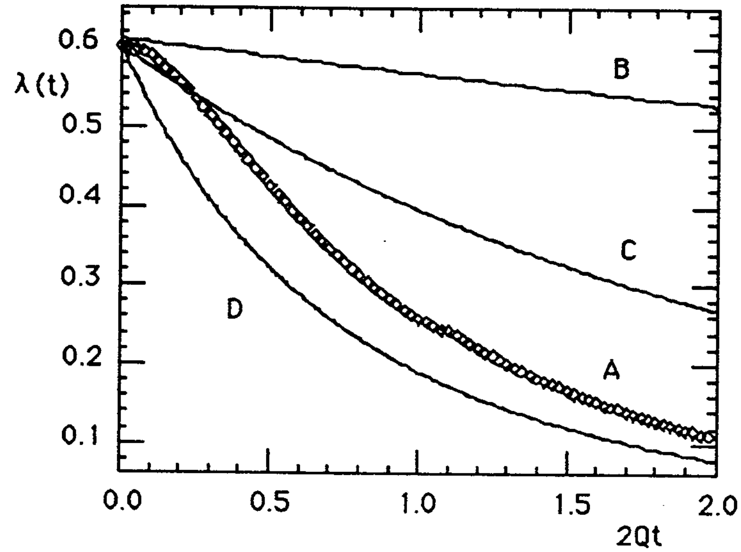

Fig. 2. Intensity correlation functions. A) Colored loss-noise model, $2Q\tau=0.3$, $\alpha=1.13$, $\beta\langle I\rangle\alpha_1 = 1.13$. Remaining lines are for the white gain-noise model: B) $\alpha= 0.32$, $\alpha_1=0.417$, $\beta\langle I\rangle = 3.31$. C) $\alpha= 0.76$, $\alpha_1=1.63$, $\beta\langle I\rangle=0.87$. D) $\alpha= 1.30$, $\alpha_1=8.0$, $\beta\langle I\rangle = 0.19$.

It has been proved[5] that when only colored noise is present, λ_{eff} is strictly zero. This is the case for the colored loss-noise model when spontaneous emission noise is neglected. As a consequence, a small initial slope of the correlation function has been interpreted as a signature of colored noise. A calculation of λ_{eff} for the white gain-noise model gives the following findings: for α close to α_1, and both not too large, λ_{eff} takes very small values, much smaller than the ones for the white noise limit of the loss-noise model. Then, a small initial slope in the correlation function should not be uniquely associated with the presence of colored noise.

A true experimental evidence of the presence of colored noise is the existence[6] of an initial plateau in the correlation function which lasts for a time of the order of the correlation time of the noise, followed by a faster decay. This is seen in Fig.2, where we compare a correlation function for the colored loss-noise model with several ones for the white gain-noise model. All these correlation functions have an initial value close to $\lambda(0)=0.6$ and they have been obtained by computer simulation. Although the white gain-noise model can give a very small initial slope, it does not reproduce the two-time scales obtained in the colored loss-noise model . The two time scale behavior seen in measurements of the initial decay of $\lambda(t)$ is the remainig feature indicating that a correct modeling of external pump noise in dye lasers must take into account a finite correlation time.

COLORED GAIN-NOISE

The characteristic feature of the decay of correlations functions with colored noise which we have discussed above appears both in a loss or gain-noise model. As a first guide to distinguish between these two models we consider a linearization approximation. The spectrum of the linearized fluctuations around the deterministic steady state intensity $I_0 = (\Gamma-\kappa)/\beta\kappa$, defined as the Fourier transform of $\lambda(t)$, can be calculated for the gain noise model from equation (1). We obtain.

$$S(\omega)=\left[4 I_0^{-1} D/\left(\omega^2+\gamma^2\right)\right] + Q^{1}\alpha^{-2}\gamma^2\tau^{-2} / \left(\omega^2+\gamma^2\right)\left(\omega^2+\tau^{-2}\right) \qquad (3)$$

where $\gamma=2\ Q\ \alpha\ \alpha_2\ /\ \alpha_1$. The first tem of the spectrum is associated with spontaneous emission noise and can be negleted. The second term is a product of two lorentzians, one is given by the spectrum of the colored gain noise and the second has a width γ. The same calculation for the loss noise-model, linearizing around the corresponding steady state intensity $I_0' = (\Gamma - \kappa)\ /\ \beta\ \Gamma$ gives[2] the same result with $\gamma' = 2Q\ \alpha$. It is clear that both spectra will be different in domains in which α_2 and α_1 are grossly different, that is $\alpha \approx \alpha_1$ ($\Gamma >> \kappa$). In particular this includes the region II of Fig. 1 in which both models are different for $\tau = 0$. Linearization has a different range of validity for the gain and loss noise models. An evidence of the differences in the correlation functions for the range of parameters just discussed is given in Fig.3., where results of a direct simulation of (1) are shown. The value of τ is the one determined in Ref.2, while Γ and κ have been arbitrarily varied to meet the requeriment $\Gamma >> \kappa$. A more interesting difference in the correlation functions of the two models arises only as a consequence of the value of τ . A guide to this result is obtained by a linearized calculation of $\lambda(0)$ which for the gain-noise model gives for D=0 :

$$\lambda(0) = Q^{-1}\ \gamma\tau^{-1}\ /\ 2\left(\gamma+\tau^{-1}\right)\ \alpha^2 \tag{4}$$

Again the same result is obtained for the loss noise model with γ replaced by γ'. For the range of parameters of Ref.2 in which $\alpha >> Q\ \tau^{-1}$ and α and α_1 of the same order, differences between the two models should occur for $\alpha_2\ Q\tau < 1$ ($\kappa\tau < 1$). Evidence for this is given in the simulation results of Fig 4. The values of the gain and loss parameters are the ones determined in Ref. 2 but τ has been changed from a value $\tau = 2.10^{-5}$ sec. such that $\kappa\tau >> 1$ to a value for which $\kappa\tau \approx 1$. It is finally interesting to note that for the values of τ, Γ and κ of Ref.2, λ (t) is very similar for the gain and loss-noise models in agreement with the above discussion. However, important differences occur in the unnormalized correlation function $C(t) = \langle I(t+t')I(t')\rangle - \langle I\rangle^2$ as shown in Fig.5.

ACKNOWLEDGMENT

The financial support of the Dirección General de Investigación Científica y Técnica (Spain), Project 86-0534 is acknowledged.

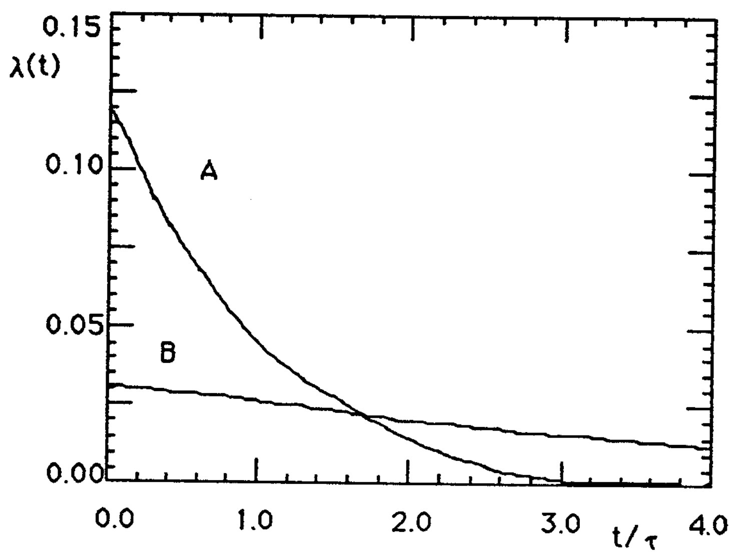

Fig. 3. Intensity correlation functions: $\Gamma=10^6\ s^{-1}$, $\kappa=10^4\ s^{-1}$, $\tau= 2\cdot10^{-5}$ s, D= $10^{-3}\ s^{-1}$, Q= $5\cdot10^{-6}\ s^{-1}$, $\beta=10^{-2}$. A) Loss-noise model, B)Gain-noise model.

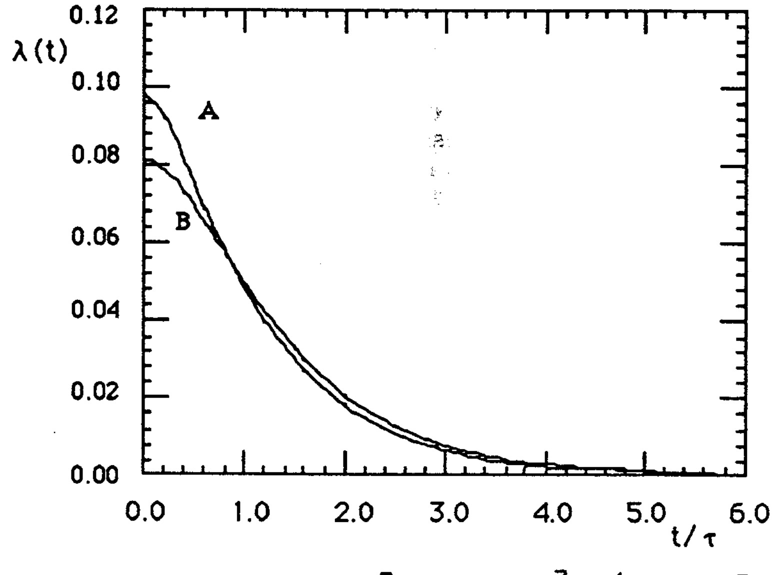

Fig. 4. Same as Fig. 3 except $\tau = 10^{-7}$ s, $\Gamma = 2 \cdot 10^{7}$ s^{-1} $\kappa = 10^{7}$ s^{-1}

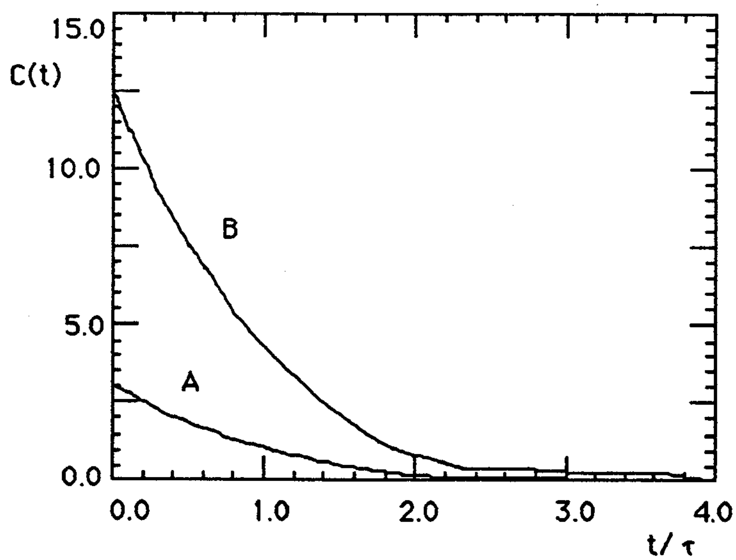

Fig.5. Unnormalized correlation functions. Same parameters as in Fig. 4 except $\tau = 2 \cdot 10^{-5}$ s A) Loss-noise model.
B) Gain-noise model.

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